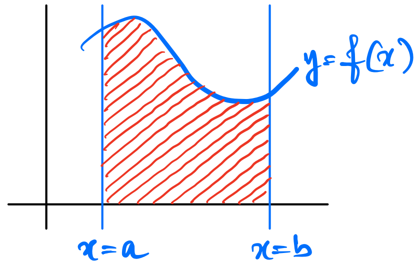
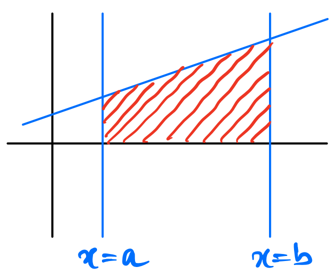
The Definite Integral:

$$\int_a^b f(x)\,dx =$$



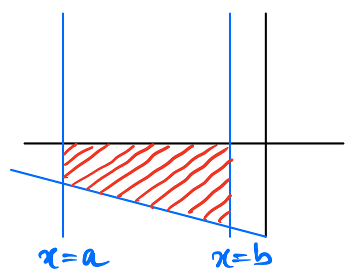
The **net signed area** between $y=f(x)$ & x-axis & bounded on left by $x=a$ & by $x=b$ on right.

Understanding Signed Areas:-



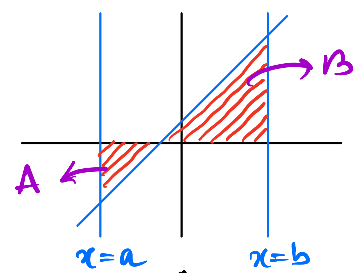
area is completely above x-axis

+ve area



area is completely below x-axis

−ve area



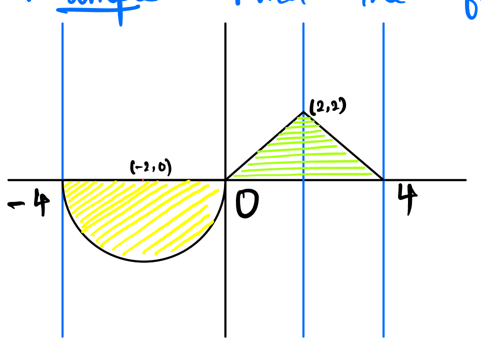
area is partially above & partially below x-axis.

A is −ve area

B is +ve area

So, total area is $(-A+B)$

Example: Find the following area:



$\Rightarrow$

semicircle (in yellow) has diameter 4 (d), so area $= \pi\cdot\left(\frac{d}{2}\right)^2$

$= \pi\cdot\left(\frac{4}{2}\right)^2$

$= \pi\cdot 4 = 4\pi$

triangle has base 4 & height 2, so area $= \frac{1}{2}(\text{base})(\text{height})$

$= \frac{1}{2}(4)(2)$

$= 4$

So required area is $(-4\pi+4)$.

Note:- Integrals are basically of two types:

① Definite Integral ⇝ measures area (a real number)
② Indefinite integral ⇝ provides an antiderivatives (a function)

Left & Right Riemann Sums:-

$\int_a^b f(x)\,dx$ ⇝
- for such problem our interval is $[a,b]$.
- divide $[a,b]$ in n-subintervals $[x_0,x_1],[x_1,x_2],\cdots,[x_{n-1},x_n]$, where $x_0 = a$, $x_n = b$
- then $\Delta x = \dfrac{b-a}{n}$.

So, Left Riemann Sum (L_n) is

$$L_n = \underbrace{f(x_0)}_{\text{left end point of } [x_0,x_1]}\cdot\Delta x + \underbrace{f(x_1)}_{\text{left end point of } [x_1,x_2]}\cdot\Delta x + \cdots + \underbrace{f(x_{n-1})}_{\text{left end point of } [x_{n-1},x_n]}\cdot\Delta x$$

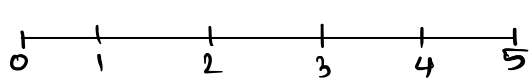
So, Right Riemann Sum (R_n) is

$$R_n = \underbrace{f(x_1)}_{\text{right end point of } [x_0,x_1]}\cdot\Delta x + \underbrace{f(x_2)}_{\text{right end point of } [x_1,x_2]}\cdot\Delta x + \cdots + \underbrace{f(x_n)}_{\text{right end point of } [x_{n-1},x_n]}\cdot\Delta x$$

Note: $\displaystyle\int_a^b f(x)\,dx = \lim_{n\to\infty} L_n = \lim_{n\to\infty} R_n$

Example: Let $f(x) = x\sqrt{x^3+1}$. Find L_5 & R_5 on the interval $[0,5]$. ($n = 5$, $a = 0$, $b = 5$)

Here, $\Delta x = \dfrac{b-a}{n} = \dfrac{5-0}{5} = 1$



So, $L_5 = \Delta x\cdot f(0) + \Delta x\cdot f(1) + \Delta x\cdot f(2) + \Delta x\cdot f(3) + \Delta x\cdot f(4)$

$R_5 = \Delta x\cdot f(1) + \Delta x\cdot f(2) + \Delta x\cdot f(3) + \Delta x\cdot f(4) + \Delta x\cdot f(5)$

Properties related to Definite Integral:

① $\int_a^a f(x)\,dx = 0$

② $\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$

③ If $a \leq c \leq b$, then $\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$

④ $\int_a^b \left(f(x) \pm g(x)\right) dx = \int_a^b f(x)\,dx \pm \int_a^b g(x)\,dx$

⑤ $\int_a^b c\,f(x)\,dx = c\int_a^b f(x)\,dx$, where c is a constant.